

Azonosító jel:

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ÉRETTSÉGI VIZSGA • 2006. május 9.

**MATEMATIKA
ANGOL NYELVEN
MATHEMATICS**

2006. május 9. 8:00

**EMELT SZINTŰ ÍRÁSBELI
ÉRETTSÉGI VIZSGA
ADVANCED LEVEL
WRITTEN EXAM**

Az írásbeli vizsga időtartama: 240 perc
The exam is 240 minutes long

Pótlapok száma/Number of extra sheets	
Tisztázati/Final essays	
Piszkozati/Drafts	

**OKTATÁSI MINISZTERIUM
MINISTRY OF EDUCATION**

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Important Information

The exam is 240 minutes long. After that you should stop working.

You may attempt the questions in arbitrary order.

You are supposed to answer four out of the five questions in part II. **The number of the question not attempted should be entered into the empty square below.** Should there arise any ambiguity for the examiner as for which question should not be marked, it is question no. 9 that will not going to be assessed.

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You may work with any calculator as long as it is not capable of storing and displaying textual information and you may also consult any type of four digit mathematical table. The use of any other kind of electronic device or written source is forbidden.

Remember to show your reasoning; a major part of the marks is given for this component of your work.

Remember to show your working, including substantial calculations.

When you refer to a theorem that has been done at school and has a common name (e.g. Pithagoras' theorem, sine rule, etc.) you are not expected to state it meticulously; it is usually sufficient to put the name of the theorem. However, you should briefly explain how the theorem applies in your solution. Any other reference to any other kind of mathematical theorem will receive full credit only if it is stated correctly (no proof is needed) and its relevance and applicability is clearly explained.

Remember to answer each question (i.e. the result) also in textual form.

You are supposed to work in pen; diagrams, however, may also be drawn in pencil. Anything written in pencil outside the diagrams cannot be evaluated by the examiner. Any solution or some part of a solution that is crossed out will not be marked.

There is only one solution will be marked for every question.

Please, do not to write anything in the shaded rectangular areas.

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I.

1. The endpoints of the base of an isosceles triangle are $A(3, 5)$ and $B(7, 1)$. The third vertex of the triangle is lying on the y -axis.
- a) Find the coordinates of the third vertex of the triangle.
- b) Write down the equation of the circumcircle of the triangle.

a)	4 points	
b)	8 points	
T.:	12 points	



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2. Given are two cubes, one of them is blue and the other one is red. The surface area of the red cube is 25% less than that of the blue one. How much less, in percentage, the volume of the red cube than that of the blue one?

12 points	
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3. The real roots of the equation $x^2 - x + p = 0$ are one less, than the real roots of the equation $x^2 + px - 1 = 0$, respectively.

- a) Find the value of the real parameter p .
b) Calculate the real roots of both equations if $p = 5$.

a)	9 points	
b)	4 points	
T.:	13 points	

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4. The members of a group of 30 scientists are investigating the applications of computers in research, education and communication. Every member of the group has already published something in at least one of these areas. There are 12 of them who have written on the role of computers in research, 18 of them have published articles on their use in education, finally 17 scientists have published studies on how computers are used in communication. There are 7 scientists in the group who have published surveys about exactly two of the above issues.
- a) One member is selected randomly from the group, to participate in a television talk-show. Find the probability that this scientist has already published some papers in each of the above research areas.
- b) How many specialists are there in the group, i. e. scientists who have published in one field only?

a)	10 points	
b)	4 points	
T.:	14 points	

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II.

You are supposed to answer any four out of the questions no. 5-9. The number of the question not attempted should be entered into the empty square on sheet no. 3.

- 5.** The roof of the tower of a medieval church from the roman period is a right pyramid with a square base. The length of each side of the base is equal to the edges of the pyramid. On renovation there was a cubical room of maximal volume fashioned inside the roof structure; its base was on a level with the base of the pyramid and the corners of its ceiling were incident to the edges of the pyramid.
- a) Find the area of the base of the room in the roof if it is given that the edges of the pyramid are 8 m long.
- b) What percentage of the airspace of the roof is filled by this room?

a)	9 points	
b)	7 points	
T.:	16 points	

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You are supposed to answer any four out of the questions no. 5-9. The number of the question not attempted should be entered into the empty square on sheet no. 3.

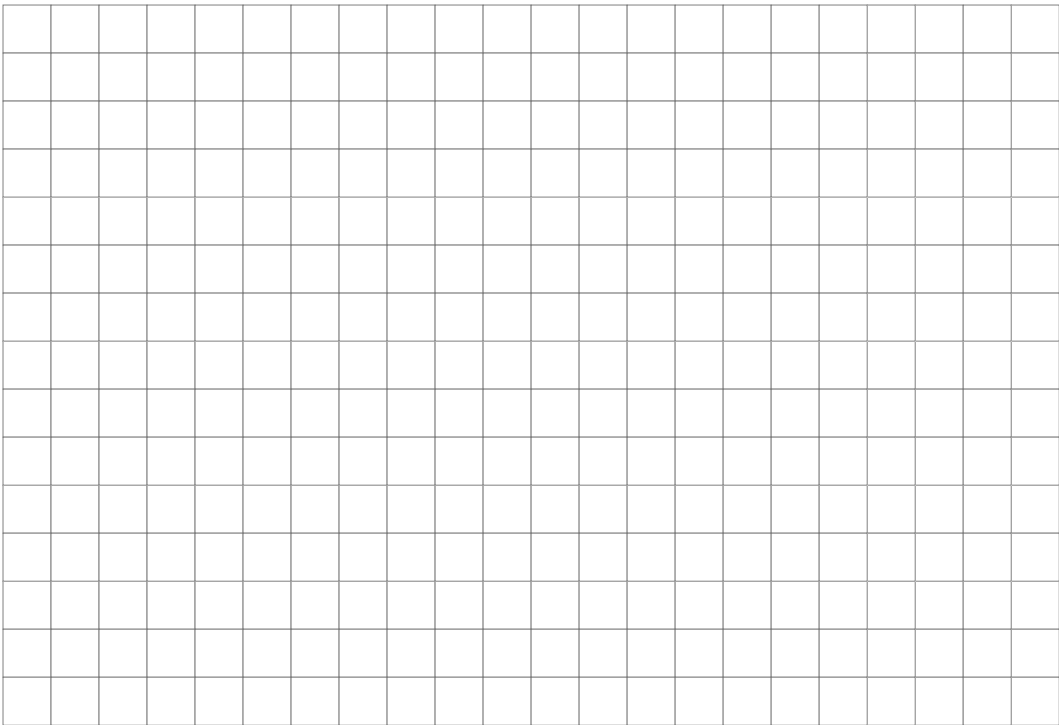
6. Given are the functions $f: \mathbf{R} \rightarrow \mathbf{R}$, $f(x) = -x^2 + 10x - 22$ and $g: \mathbf{R} \rightarrow \mathbf{R}$, $g(x) = -x + 6$.

- a) Solve the equation $f(x) = g(x)$.
- b) Write down the equations of the tangents to the curve of equation $y = f(x)$, at the common points of the two curves whose equations are $y = f(x)$ and $y = g(x)$, respectively.
- c) Sketch the graphs of the functions f and g . The curves whose equations are $y = f(x)$ and $y = g(x)$, respectively and the line $x = 6$ enclose two regions. Calculate the area of the one that is lying closer to the y-axis.

a)	3 points	
b)	7 points	
c)	6 points	
T.:	16 points	



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You are supposed to answer any four out of the questions no. 5-9. The number of the question not attempted should be entered into the empty square on sheet no. 3.

7. Because of railway restorations the train from Szeged to Budapest was forced to travel between Cegléd and Budapest by one third of its scheduled average velocity. On the weekend, however, it could still attain its original average velocity along a 19 km long line right after Cegléd, but beyond that it could travel again by one third of its average velocity only. The delay of the train hence was 30 minutes more on Monday than that on the weekend.

- a) What is the average velocity of this train when it is on schedule?

When scheduling its budget the MÁV (National Railway Company) is frequently carrying out surveys among its passengers on the distribution of discounts and that of the tickets purchased.

On a particular Budapest- Szeged trip there were 400 passengers traveling altogether from Budapest to Szeged (i. e. between the two termini). There were only second class coaches on the train and the full price of a 2nd class ticket for the whole trip was approx. 2 000 forints. (To make things simpler we shall refer to this rounded price onwards.) The controllers noted, for every passenger the price of its ticket and the eventual discount. The data is indicated in the table below. (A ticket used to be called of $x\%$ discount if the actual price is the total price reduced by $x\%$.)

Type of the ticket	Full price	20% discount	33% discount	50% discount	67,5% discount	75% discount	90%- discount	95% discount	Free
No. of passengers	84	18	44	110	11	35	31	29	38
Actual Price (Ft)									

- b) Complete the table and determine the discount corresponding to the average price.

a)	10 points	
b)	6 points	
T.:	16 points	

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- 8. a)** When written in decimal form, the one digit number \bar{a} , the two digit number $\bar{a}\bar{b}$ and the three digit number $\bar{b}\bar{b}\bar{a}$ are the first three terms, in this order of an arithmetic progression (Identical letters stand for equal numbers and different letters for different numbers.) Find the common difference of this arithmetic progression and calculate the sum of its first one hundred terms.
- b)** Prove that the sums of the first n terms, the second n terms and the third n terms, respectively, of a geometric progression are the three consecutive terms of a geometric progression.

a)	7 points	
b)	9 points	
T.:	16 points	

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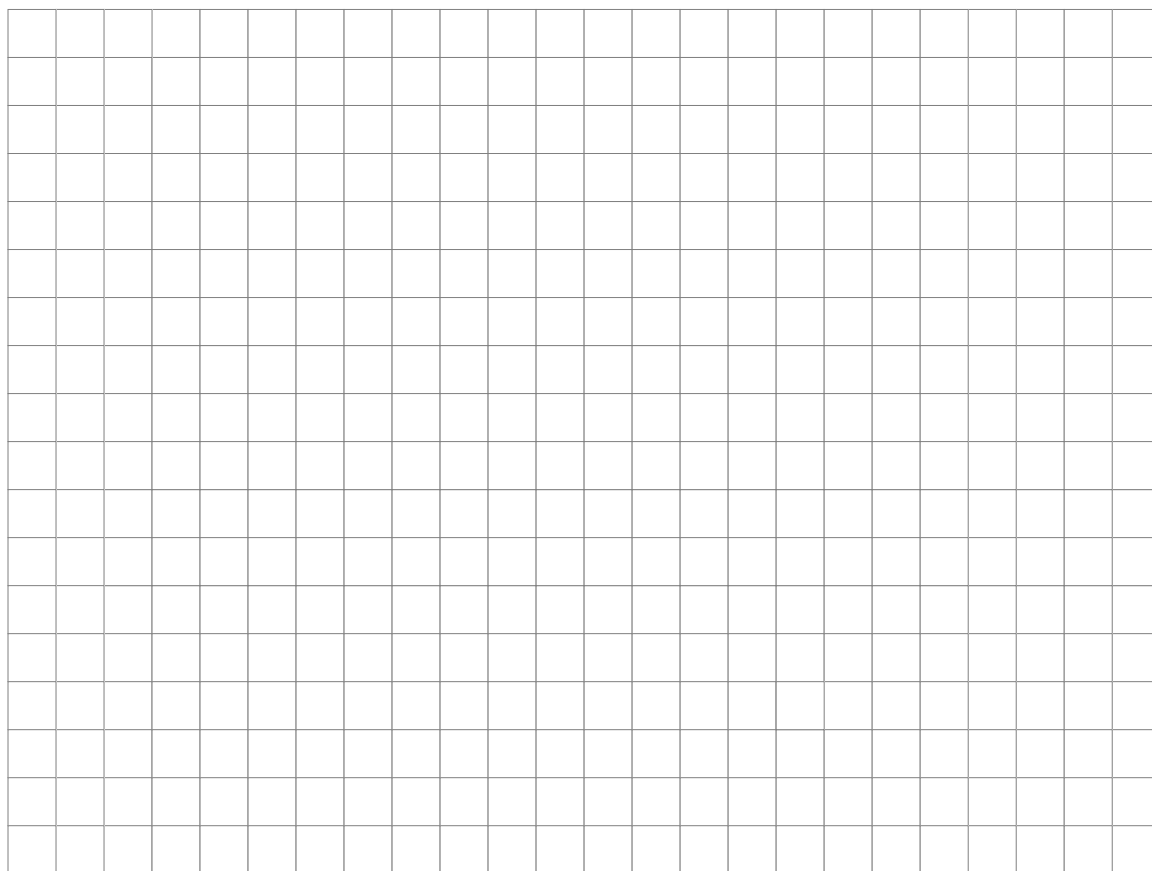
9. The board of trustees of the foundation of a certain high school is organizing a lottery. The income should be devoted partly to the prizes and partly to charity purposes. There are four numbers drawn randomly in this lottery from the first 40 positive integers every week. Andrew has a particular method of filling his ticket: having selected the first two numbers he marks the sum of these numbers as the third one and the sum of the first three numbers as the fourth one.
- a) What is the maximum value of the smallest number on a ticket that is filled according to the preceding method?
- b) If Andrew chooses the highest possible value of his smallest number then, according to his method, which numbers will be marked on his ticket?
- c) On a particular week Andrew decides to play every single ticket obeying his rule exactly once. What is the probability that he will eventually hit the jackpot?

a)	4 points	
b)	4 points	
c)	8 points	
T.:	16 points	

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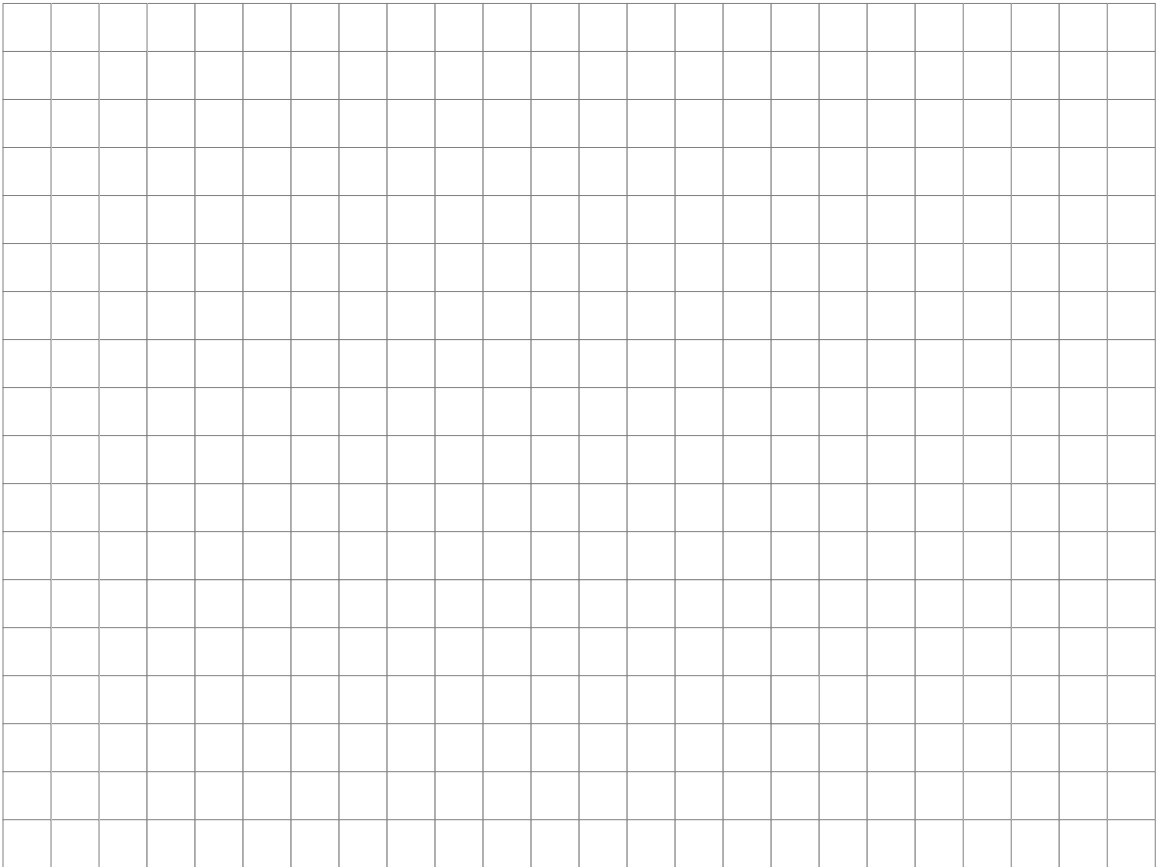
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(You may also prepare sketches or solutions on this sheet.)



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(You may also prepare sketches or solutions on this sheet.)



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	Number of question	score	total	maximal score
PART I.	1.			12
	2.			12
	3.			13
	4.			14
PART II.				16
				16
				16
				16
		← problem not chosen		
TOTAL				115

_____ date

_____ examiner

	a feladat sorszáma/ Number of question	elért pontszám/ score	programba beírt pontszám/ score written in the programme
PART I.	1.		
	2.		
	3.		
	4.		
PART II.			

_____ dátum/date

_____ javító tanár/Examiner

_____ Jegyző/Registrar